DISSOLVING OF A CHAIN OF DROPS (BUBBLES) IN A FLUID STREAM

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The simplest model problem of the dissolving of a chain of drops (bubbles) over which a stream of viscous incompressible fluid flows at low Reynolds numbers is considered. The diffusional interaction of the drops due to the presence of diffusional tracks [1, 2] is allowed for. The radii of the drops and the dissolving rate are determined as a function of their position in the chain and time; the characteristic dissolving times of the drops are obtained. It is shown that the diffusional interaction in chains leads to significant slowing of the dissolving process. For a drop with the serial number k the total dissolving time t_k is determined by the formula

$$t_k = \alpha\sqrt{k}, \quad \alpha = \text{const},$$

where the numbering is made from the drop traveling at the front.

We assume that the drops (bubbles) move in the fluid one after another with a constant velocity U and retain a spherical shape in the process of dissolving, while at the initial time they have equal radii $a_k^*(0) = a$ (k = 1, 2,..., M); the initial Reynolds number $\mathrm{Re} = aU\nu^{-1}$ is small, while the Peclet number $\mathrm{Pe} = aUD^{-1}$ is large (ν is the coefficient of kinematic viscosity and D is the coefficient of diffusion).

The initial distribution of the fluid velocities and the concentration is determined by the joint solution of the steady-state Stokes and convective-diffusion equations with boundary conditions of constancy of the fluid velocity U and the concentration c_0 far from the chain, as well as the appropriate dynamic conditions and the condition of equality of the concentration c_1 at the surfaces of the drops [3]. We assume that the dissolving process is isothermal, while the concentration of the substance inside and at the surfaces of the drops is a constant value independent of time or the number of drops.

The kinetics of the simple (physical) dissolving is determined by the process of convective diffusion of material toward the surfaces of the spheres [3] and is given by the mass-conservation law $dm_k/dt = I_k$, where dm_k/dt is the total change in the mass of the k-th drop per unit time; I_k is the total diffusion flux to its surface. Substitution of the steady-state value I_0 for a single sphere (which, because of the large Peclet numbers, is determined from the solution of the corresponding diffusional boundary-layer problem [3]) into the conservation law shows that the characteristic time of variation of the drop radius is large and on the order of $aU^{-1}\sqrt{\mathrm{Pe}}$. Therefore, the process of convective flow over a chain of drops and of its convective diffusion is quasi-steady and the radii $a_k^* = a_k^{*(t)}$ of the drops vary slowly with time. This means that the procedure for solving the problem can be reduced to three successive stages: 1) construction of a steady-state solution of the Stokes and convective-diffusion equations for a chain of drops with radii a_k^*; here a_k^* are not fixed and play the role of parameters; 2) calculation of the total diffusion fluxes $I_k = I_k(a_1^*, a_2^*, \ldots, a_k^*)$ to the surfaces of the drops from the solution of the preceding problem; 3) substitution of the resulting fluxes I_k into the mass-conservation equations, which gives an independent nonlinear system of ordinary differential equations for determining the drop radii with the initial condition $a_k^*(0) = a$.

The first stage of the solution of the total problem is the most complicated, and its solution in the general case is unknown. It should be noted, however, that the fluid velocity field for a chain consisting of two drops of arbitrary radius (M = 2) was obtained in [4-6], while the concentration field can also be constructed with the help of [1, 2].

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For simplicity we assume in future that the initial distances $l_k = l$ between drops satisfy the inequality

$$aO(1) < l < aO(\sqrt{\mathrm{Pe}}). \tag{1}$$

The The left side of the inequality allows one to use the Stokes solution for a single drop,

$$\psi_k = \frac{1}{2} U (r_k - a_k^*) \left[r_k - \frac{1}{2} \frac{\beta}{\beta + 1} a_k^* \left(1 + \frac{a_k^*}{r_k} \right) \right] \sin^2 \theta_k, \tag{2}$$

to determine the velocity field in the vicinity of the k-th drop, with the accuracy of the lowest terms, where β is the ratio of the viscosity of the drop ($\beta_k = \beta$) and that of the surrounding fluid, r_k, θ_k is the spherical coordinate system connected with the center of the k-th drop; the polar angle θ_k is measured from the stream direction (the trailing critical point).

To calculate the total diffusion flux to the k-th drop,

$$I_k = I_\Sigma^{(k)} - I_\Sigma^{(k-1)} = 2\pi a_k^{*2} D \int_0^\pi \left[\frac{\partial c_k}{\partial r_k} \right]_{r_k = a_k^*} \sin \theta_k d\theta_k, \tag{3}$$

where $I_\Sigma^{(k)} = I_\Sigma^{(k)} (a_1^*, a_2^*, \ldots, a_k^*)$ is the total diffusional flux to the first k drops of the chain, one must obtain the distribution of the concentration c_k in the diffusional boundary layer of the k-th drop, which is determined from the solution of the steady-state equation

$$\frac{\partial \psi_k \partial c_k}{\partial \theta_k \partial r_k} - \frac{\partial \psi_k \partial c_k}{\partial r_k \partial \theta_k} = D \sin \theta_k \frac{\partial}{\partial r_k} \left(r_k^2 \frac{\partial c_k}{\partial r_k} \right) \tag{4}$$

with boundary conditions of constancy of the concentrations of the substance far from the drop ($c = c_0$) and at the surface of the drop ($c = c_1$) and the onflow condition (the boundary condition for the concentration taken in the diffusional boundary layer of the k-th drop).

The right side of the inequality (1) indicates that the case under consideration corresponds to the location of the k-th drop in the convective boundary-layer region of the diffusional track of the preceding (k − 1)-th drop [1, 2]. This in turn means that in the vicinity of the leading critical point of the k-th drop the concentration must be set equal to the concentration "at the exit" from the diffusional boundary layer of the (k − 1)-th drop [1, 2].

The solution of the problem (2), (4) with the indicated boundary conditions and arbitrary values of the drop radii was obtained in [2], and for the total diffusional fluxes it leads to the equality

$$I_\Sigma^{(k)} = 4 (c_1 - c_0) \sqrt{\frac{2\pi}{3(\beta + 1)}} U^{1/2} D^{1/2} \left[\sum_{i=1}^{k} (a_i^*)^3 \right]^{1/2}. \tag{5}$$

Using the law of conservation of mass and Eqs. (3) and (5), and also considering that $m_k = {}^4/_3 \pi a_k^{*3}$, we obtain the following law determining the kinetics of the dissolving of drops of a chain:

$$\frac{1}{2} \frac{d}{d\tau} (a_k^3) = - \left(\sum_{i=1}^{k} a_i^3 \right)^{1/2} + \left(\sum_{i=1}^{k-1} a_i^3 \right)^{1/2}, \quad a_k(0) = 1, \tag{6}$$

$$\tau = \lambda \frac{t}{T}, \quad \lambda = \frac{1}{2} \sqrt{\frac{6}{\pi}} (\beta + 1)^{-1/2}.$$

Equation (6) is written in dimensionless variables, where a, $(c_0 - c_1)$, U, and $T = aU^{-1}\sqrt{Pe}$ are chosen as the scales of the particle radii, the concentration, the velocity, and time, respectively.

We sum Eq. (6) from 1 to k,

$$dJ_k/d\tau = -2\sqrt{J_k}, \quad J_k(0) = k, \quad J_k = \sum_{i=1}^{k} a_i^3(\tau), \tag{7}$$

and after integration we obtain

$$J_k(\tau) = (\sqrt{k} - \tau)^2. \tag{8}$$

Using (8) we determine the law of time variation of the drop radii:

$$a_1(\tau) = (1 - \tau)^{2/3}, \; k = 1, \tag{9}$$
$$a_k(\tau) = [J_k(\tau) - J_{k-1}(\tau)]^{1/3} = [1 - 2(\sqrt{k} - \sqrt{k-1})\tau]^{1/3}, \; k \geqslant 2.$$

From the moment the first drop dissolves ($\tau_1 = 1$, $a_1(\tau_1) = 0$) the second drop becomes the first drop of the chain and, as follows from (9), at this time its radius is almost twice as small as the initial radius: $a_2(\tau_1) = (\sqrt{2} - 1)^{2/3} = 0.554$. Equation (8) gives the initial condition for $J_k(\tau)$ at $\tau = \tau_1$, so that the kinetics of the dissolving of a chain at $\tau_1 \leqslant \tau \leqslant \tau_2$, $a_2(\tau_2) = 0$, is determined by the system

$$dJ_k/d\tau = -2\sqrt{J_k}, \; J_k(\tau_1) = (\sqrt{k} - 1)^2, \quad J_k = \sum_{i=2}^{k} a_i^3(\tau). \tag{10}$$

Similarly, by obtaining the solution of (10) and determining τ_2 and $J_k(\tau_2)$ one can write the equations determining the kinetics of the system after the second drop dissolves, and so forth.

Omitting the intermediate calculations, here we present only the final, most important results, namely, the time τ_k when the k-th drop dissolves and the radius $a_{k+1}(\tau_k)$ of the next (k + 1)-th drop at this time:

$$\tau_k = \sqrt{k}, \; a_{k+1}(\tau_k) = (\Delta\tau_{k+1})^{2/3} = (\sqrt{k+1} - \sqrt{k})^{2/3}. \tag{11}$$

From this it is seen that the time interval between two successive dissolvings approaches zero at large k: $\Delta\tau_k = \tau_k - \tau_{k-1} = \sqrt{k} - \sqrt{k-1} \cong 0.5k^{-1/2}$.

As seen from (11), the presence of a diffusional interaction in chains leads to considerable slowing of the process of dissolving of the drops; in particular, the second drop takes almost one and one half times longer to dissolve than the first, and at tne moment the first one dissolves its radius is still half as large as the initial radius.

Let us indicate the region of applicability of the results obtained: 1) the results lose applicability when the diffusional boundary-layer approximation (4) becomes invalid, i.e., when the sizes of the drops decrease so much that $a_k(\tau)\sqrt{Pe} = O(1)$ is satisfied ($a_k(\tau)$ is the ratio of the current radius to the initial radius); 2) Eq. (6) is valid only so long as the condition

$$a^{-1}l < a_k(\tau)O(\sqrt{Pe_k}), \; Pe_k = a_k(\tau)\,Pe$$

is satisfied, i.e., so long as the k-th drop is in the convective boundary-layer region of the diffusional track of the preceding (k − 1)-th drop [1, 2]. Violation of the latter condition means that the drop began to enter the region of mixing of the diffusional track of the preceding drop, and this, as follows from [1-2], leads to the appearance of a multiplier $\gamma_k(\tau) < 1$ to a_k in Eqs. (6).

Both restrictions are connected with the radius of any drop becoming sufficiently small. Because the initial Peclet number is large, however, the system (6) well describes the time dependence of the radius of an arbitrary k-th drop so long as $a_k(\tau)$ does not become too small, in which case the smallness of the preceding drops (of radii) $a_1(\tau), \ldots, a_{k-1}(\tau)$ has a weak effect on the behavior of $a_k(\tau)$. When $Pe \sim 10^5$-10^6 (with $al^{-1} \simeq 0.2$), e.g., the radius of the first drop must decrease by several tens of times in comparision with the initial radius in order for the indicated restrictions to be satisfied. At this time, how-

ever, the radii of the subsequent drops exceed the radius of the first drop by more than an order of magnitude, so that for $k \geqslant 2$ the term $a_1(\tau)$ in the equations of the system (6) can be neglected as small. The same thing can be said in an investigation of the behavior of the system near $\tau \approx \tau_2$ and so forth, up to k which are not too large (the error grows in proportion to k). This indicates that the restrictions are reflected rather weakly in the final results for the radii $a_{k+1}(\tau_k)$, where τ_k is determined from Eq. (11).

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DRIPPING OF A LIQUID FROM A POINT

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The drop discharge of a liquid under the action of gravity is frequently encountered in nature, and is used in technology, e.g., in medicine droppers. In a slow drop discharge the drops formed are of approximately the same size. The formation of drops in the discharge of a liquid from a vertical stationary capillary has been investigated [1, 2]. The dripping of a liquid from a point has received little attention.

The formation of drops in the slow dripping of a liquid from the point of a vertical stationary needle whose surface is well wetted by the liquid is shown schematically in Figs. 1 and 2. The liquid flows by gravity from the reservoir 1 through the annular slit 2, and wets the conical surface of the needle of height H. The thickness of the layer of liquid increases after it flows through the slit, but not uniformly; visual observation shows that the liquid does not begin to accumulate at the point, but at a certain height h above it, in the form of a bulging collar (Fig. 2a). The thickness of the collar increases, it drops down (Fig. 2b), and gradually takes the form of a drop (Fig. 2c) which descends further to the point itself (Fig. 2d) and then quickly drips off the point (Fig. 2e) and falls downward (Fig. 2f).

In order to explain this dripping process, we consider the pressure distribution resulting from the surface tension of the liquid within a thin layer on the surface of a conical needle.

Forces due to the surface tension σ act on an annular element of the surface of a conical film of height dz at a height z (Fig. 3). The resultant of the vertical components of these forces is

$$[2\pi(R + dR)\sigma - 2\pi R\sigma] \cos\varphi = 2\pi\sigma \cos\varphi dR.$$

The resultant of the vertical components of the forces on this element as a result of the pressure p is $pS \sin\varphi$, where $S = \pi d\ell(2R+dR)$ is the area of the surface of the annular element (frustum of the cone), or, neglecting second order quantities, $S = 2\pi RdR/\sin\varphi$.

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